Non-Linear Thinking

Which of the following two options would you prefer?

A. $2,000,000
B. The result of doubling 1 cent every day for thirty days

We have given this thought experiment to a number of people and a large majority of them have opted to go with the first option. And to their hypothetical detriment, that option leaves some money on the table.

You were probably smart enough to go with the second option, though. Good for you. Now, we'll make it a bit more interesting. Let's change option A from $2 million to $5 million. Now, which of the two do you prefer?

It's a bit harder now, isn't it? Well, option B happens to be the best option again.

Let's finish with one final offer:

Which of the following two options would you prefer?

A. $10,000,000
B. The result of doubling 1 cent every day for thirty days

If you find a calculator and crunch the numbers you will see that if you start with one cent and you double it thirty times you end up with just over $10.7 million ($10,737,418.24 to be exact).

Why is it that most people find it so difficult to figure out that doubling a penny thirty times yields almost $11 million? It has something to do with nonlinear relationships.

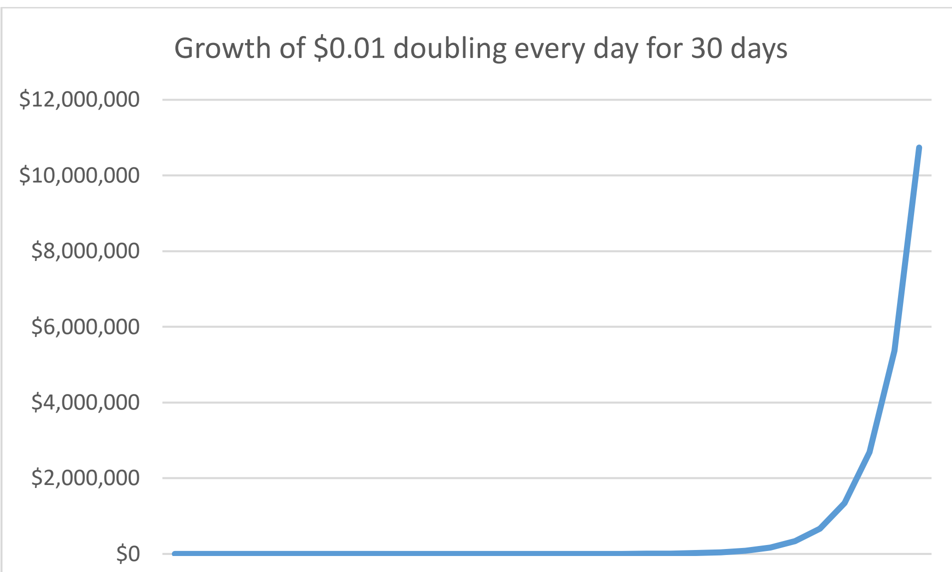
Linear vs. Nonlinear

In deciding which is the better choice in the experiment above, most people start doing a calculation in their head: one cent, two cents, four cents, eight cents, sixteen cents, thirty-two cents… They then, erroneously, come to the quick conclusion that there's no way they can get to $2 million. Or $5 million. And forget about $10 million.

The reason is that they're thinking that the amount of money in option B will grow in a straight line; however, it does so in a nonlinear way. The numbers start rising gradually, and then they rise very steeply.

Let's look at it in graphical form to visualize it better. In thinking about option B above, most people imagine a linear relationship like this:

In reality, it looks like this:



To further illustrate the difference between linear and nonlinear relationships, let's look at a couple of thought experiments that we recently came across in a [*Harvard Business Review*](#) [article](#):

1) *Your company has two factories. Assuming that the factories operate 24/7, what would give you the largest increase in the number of products made per year?*

   A. *Increase the productivity of the first factory from 100 to 120 products per hour.*
   B. *Increase the productivity of the second factory from 130 to 140 products per hour.*

2) *Imagine you're responsible for your company's car fleet. You manage two models, an SUV that gets 10 miles to the gallon and a sedan that gets 20. The fleet has equal numbers of each, and all the cars travel 10,000 miles a year. You have enough capital to replace one model with more fuel-efficient vehicles to lower operational costs and help meet sustainability goals.*

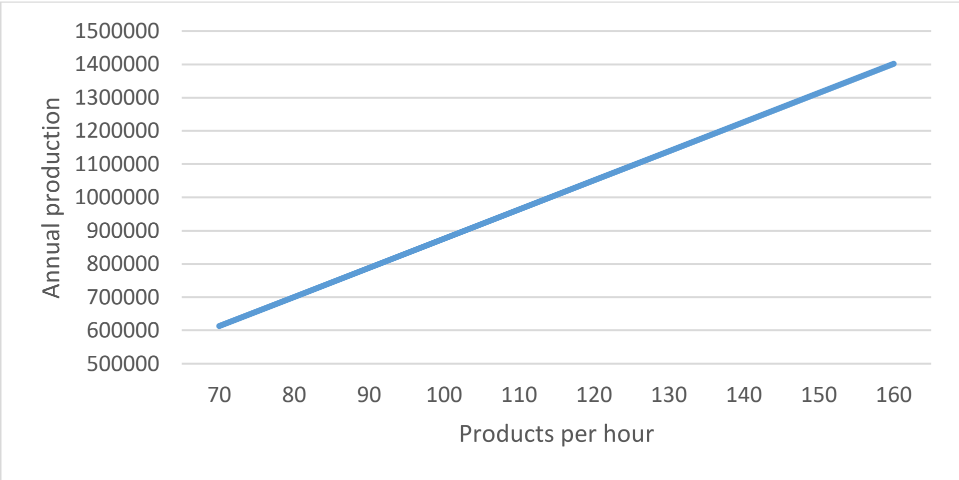
   Which upgrade is better?

   A. Replacing the 10 MPG vehicles with 20 MPG vehicles
   B. Replacing the 20 MPH vehicles with 100 MPG vehicles

Turning to the first problem, the chart below shows the products made in a year by the two factories, before and after the productivity increases.

	Current	**After Productivity Increase**	**Increase**
Factory 1	876,000 (@ 100 products/hour)	1,051,200 (@120 products/hour)	175,200
Factory 2	1,138,800 (@130 products/hour)	1,226,400 (@140 products/hour)	87,600

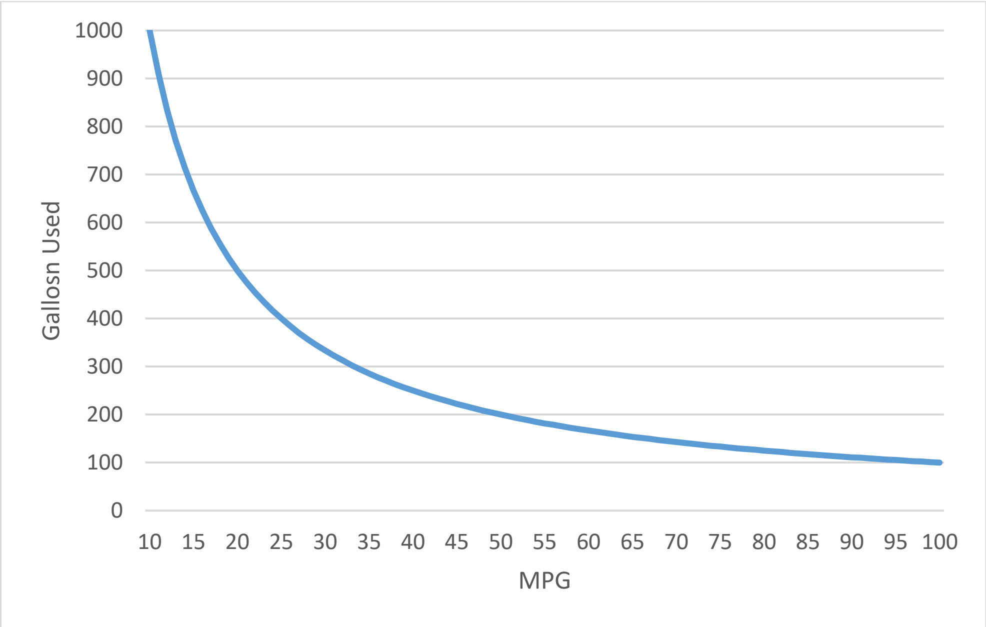
We can see that Factory 1 increased its output by double the amount of Factory 2. This is pretty straightforward: the 20 products/hour increase of the first factory (i.e., 120 – 100) is twice the 10 products/hour increase of the second factory (i.e., 140 – 130), so the annual increase in production is also double. That is a linear relationship, and it makes intuitive sense. Here it is in graphical format:



The second problem, though, is not as simple. The chart below shows the number of gallons used per 10,000 miles for each SUV and sedan before and after the upgrade.

	Current	After Upgrade	Savings
SUV	1,000 (@10 MPG)	500 (@20 MPG)	500
Sedan	500 (@20 MPG)	100 (@100 MPG)	400

Most people find these results surprising. After all, an increase from 20 MPG to 100 MPG is a lot larger than an increase from 10 MPG to 20 MPG. So how is it that going from 10 MPG to 20 MPG is the better deal? The graph below illustrates the answer by making it clear that at lower MPGs the number of gallons used falls sharply, while it falls gradually further along the curve.

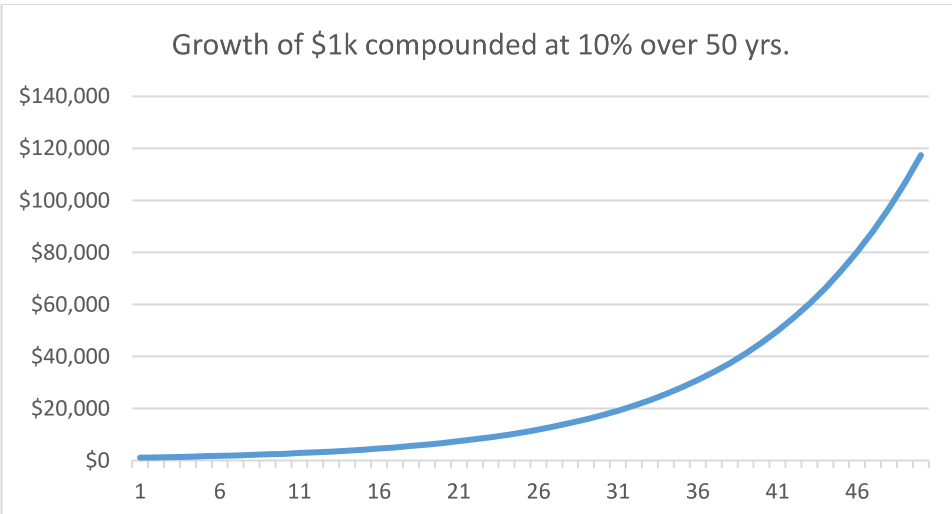


Compounding is Nonlinear

This “linear bias” is evident in many facets of business life. The *Harvard Business Review* article that we mentioned above provides examples of nonlinearity in other decision-making situations such as measuring consumer attitudes and performance metrics, as well as in how to optimally price products.

Personal finance, investing, and the stock market are also full of nonlinear relationships, and none is more evident and important than the concept of compounding. The scenario at the beginning of this article shows the magic of compounding, where gains are reinvested thus producing a snowball effect.

We don't need to such a drastic example to make our point, though. After all, nobody is going to be doubling their money every day or every year. But even if we look at the historical return of the stock market of about 10% per year, we can see the difference it makes if we invest for a long enough time.



The graph shows the growth of a $1,000 investment over 50 years if it gains 10% per year. As we have explored above, this is another example of a nonlinear relationship where the increase in wealth is gradual at the beginning and then it rises more steeply. This graph should be shown to every high school and college student in the country. There is no better argument for the advantages of starting to save and invest early in life.

Conclusion

The human brain has a hard time envisioning nonlinear relationships; it wants to view the world in simple straight lines. But there are many decisions in life that need to be made with the understanding of nonlinear relationships, and investing is one of them. We need to familiarize ourselves with this concept so that we can recognize them when they arise.

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